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Equity Derivatives

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Mr. John Hiatt
Director
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400 South La Salle
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Dear Joe,

Thank you for giving us the opportunity to discuss our suggestions for a reformulated VIX index last week. As we mentioned, we believe that the VIX index and its users would be best served if the index could become a more standardized calculation, and if it became easier to hedge using existing listed options contracts. We believe that with some relatively minor changes the calculation could become a more standard measure of implied volatility, and that with some slightly more complex modifications, the index could become hedgeable, thereby increasing the likelihood of traded products on the VIX becoming actively traded.

Our suggestions are as follows:

- i. **Move the calculation from OEX (S&P 100) to SPX (S&P 500) options prices.** Since the VIX was originally formulated, the SPX has increasingly gained market share at the expense of the OEX. Over the first quarter of 2003, our calculations suggest that volume in OEX options comprised only 12% of US index option dollar volume, while SPX options accounted for 83% of volume. Furthermore, with liquid traded futures available only on the SPX, then replicating the VIX becomes much easier when it is based on the SPX.
- ii. **Remove the business days to calendar days transformation from the calculation.** This transformation, as discussed in the original Whaley research paper¹, creates a predictable, but unstable relationship between quoted implied volatility and the level of the VIX index. Whatever the merits of the calculation, it increases instability in the VIX. Simply removing this component to the calculation will make the VIX more widely understood than it currently is. Appendix A illustrates the relationship between the VIX and quoted implied volatility as days to option expiration elapse.
- iii. **Use continuous time rather than discrete daily changes in time to expiration.** Most market makers view time to expiration as moving continuously. In other words, time is measured in days, hours and minutes to expiration. As minutes and hours elapse to expiration, time is assumed to be passing. If only the number of days to expiration is used, then the amount of time to expiration is

¹ "Derivatives on Market Volatility: Hedging Tools Long Overdue", Robert E. Whaley, Journal of Derivatives 1 (Fall 1993), 71-84

assumed to be the same in the morning as in the afternoon of a particular day. While this is a minor improvement, we believe it reflects standard practice in the options industry.

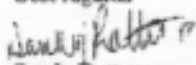
- iv. **Interpolate variance rather than volatility to calculate a standardized one month implied volatility measure.** The notion that volatility increases proportional to the square root of time, or that variance increases proportional to time, is central to the Black-Scholes formula and other option pricing techniques. The current VIX calculation, interpolates volatility between the nearby and second nearby options expirations to derive a standardized one month volatility measure. This implicitly assumes that volatility increases with time, which is inconsistent with option pricing models as well as term structure models for implied volatility.
- v. **Base the calculation of the VIX on variance swap or "fair" volatility.** Fair volatility is typically quite close to At-The-Money implied volatility (typically greater by a factor of 1.07 with a range of 1.02 to 1.12 for one month options). The major advantages of moving the calculation to fair volatility from ATM implied volatility are:
- Products based on the square of the fair volatility are hedgeable using an appropriately weighted static strip of options (options at different strikes, but the same maturity). This automatically means that options market makers will be in a better position to make markets on the VIX².
 - Fair volatility is calculated from option prices rather than implied volatility. Therefore the assumptions used in calculating implied volatility (in particular the option pricing model) are not required to calculate fair volatility.
 - Fair volatility is calculated using a range of options prices at different strikes. Therefore fair volatility captures the skew, and is also less dependent on individual ATM options prices.

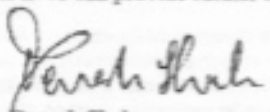
We believe that fair volatility can be approximated with quite a simple formula -- see Appendix B for the details. While we appreciate that fair volatility is a more complex concept than ATM implied volatility, we believe that producing an index which can be easily hedged (or at least whose square can be hedged) has strong advantages over other formulations of the VIX. We also note that users of the VIX have accepted the current calculation with its rather unusual relationship to quoted implied volatility and that the current calculation itself complex. Users may not be particularly concerned about the nuances of fair volatility to ATM implied volatility. Finally we believe that having one traded product (based on the VIX²) is a much neater solution than having two products, one based on the current VIX, widely followed, but difficult to hedge, and the other based on variance swaps, easier to hedge but with relatively lower following.

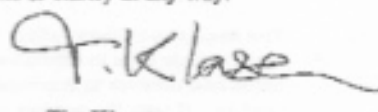
- vi. **Make any products based on the VIX based on the square of the index.** Products based on volatility (standard deviation) are extremely difficult to hedge. In contrast, products based on variance can be hedged with a static strip of options. The best definition of variance for hedging purposes is fair variance, as described in Appendix B. However, even if it is not possible to use the fair variance calculation, based on the square of ATM implied volatility would be substantially easier to hedge than products based on ATM implied volatility itself.

We believe that each of these modifications would improve the likelihood of success of contracts based on the VIX index. Please do contact us if we can provide further details or clarify in any way.

Best regards

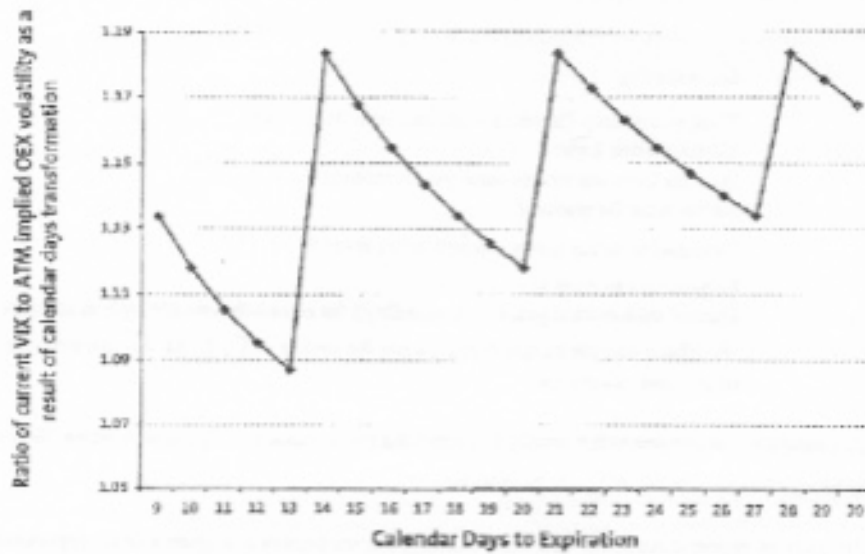

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Appendix A: Relationship of current VIX volatility to one month ATM implied volatility due to calendar day adjustment

If implied volatility on the OEX ATM nearby and second nearby options were fixed, the current VIX calculation would cause the VIX to have a level above OEX ATM volatility by a factor illustrated below. This is a result of the calendar day adjustment.



Appendix B: Calculation of Variance Swap, or "Fair" Volatility

$$\sigma_{fair}^2 = \frac{2}{T} \sum_i \frac{\Delta K}{K_i^2} e^{rT} P_{OTM}(K_i) - \frac{1}{T} \left(\frac{e^{(r-d)T} S}{K_0} - 1 \right)^2$$

Where:

σ_{fair}	is	fair volatility
T		Time to maturity (in years – i.e. calendar days / 365)
S		Current index level
ΔK		Spacing between strikes used for calculation
K_i		Strike price for option i
K_0		First option strike below current index level S
R		Interest rate to time T
$P_{OTM}(K_i)$		Quoted mid-market price (not volatility) for out-of-the-money option at strike K_i (i.e. a call for strikes $> K_0$, a put for strikes $< K_0$). At K_0 use average of put and call prices.

Given σ_{nearby} calculated for options at the nearby (T_1) and the second nearby (T_2) option terms, then we can linearly interpolate in time (t) $\sigma_{nearby}^2 \cdot t$ to calculate $\sigma_{nearby}^2 \cdot 30/25 \cdot t$

This calculation is an approximation for true fair volatility, but we believe it is quite a close approximation. We would recommend using a total of eight strike prices (four puts and four calls), currently with a separation of 25 index points.