PRIVATE AND CONFIDENTIAL

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Mr. John Hiatt
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Dear Joe,

Thank you for giving us the opportunity to discuss our suggestions for a reformulated VIX index last week. As we mentioned, we believe that the VIX index and its users would be best served if the index could become a more standardized calculation, and if it became easier to hedge using existing listed options contracts. We believe that with some relatively minor changes the calculation could become a more standard measure of implied volatility, and that with some slightly more complex modifications, the index could become hedgeable, thereby increasing the likelihood of traded products on the VIX becoming actively traded.

Our suggestions are as follows:

i. Move the calculation from OEX (S&P 100) to SPX (S&P 500) options prices. Since the VIX was originally formulated, the SPX has increasingly gained market share at the expense of the OEX. Over the first quarter of 2003, our calculations suggest that volume in OEX options comprised only 12% of US index option dollar volume, while SPX options accounted for 83% of volume. Furthermore, with liquid traded futures available only on the SPX, then replicating the VIX becomes much easier when it is based on the SPX.

ii. Remove the business days to calendar days transformation from the calculation. This transformation, as discussed in the original Whaley research paper, creates a predictable, but unstable relationship between quoted implied volatility and the level of the VIX index. Whatever the merits of the calculation, it increases instability in the VIX. Simply removing this component to the calculation will make the VIX more widely understood than it currently is. Appendix A illustrates the relationship between the VIX and quoted implied volatility as days to option expiration elapse.

iii. Use continuous time rather than discrete daily changes in time to expiration. Most market makers view time to expiration as moving continuously. In other words, time is measured in days, hours and minutes to expiration. As minutes and hours elapse to expiration, time is assumed to be passing. If only the number of days to expiration is used, then the amount of time to expiration is

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assumed to be the same in the morning as in the afternoon of a particular day. While this is a
minor improvement, we believe it reflects standard practice in the options industry.

iv. Interpolate variance rather than volatility to calculate a standardized one month implied
volatility measure. The notion that volatility increases proportional to the square root of time, or
that variance increases proportional to time, is central to the Black-Scholes formula and other
option pricing techniques. The current VIX calculation, interpolates volatility between the nearby
and second nearby options expirations to derive a standardized one month volatility measure.
This implicitly assumes that volatility increases with time, which is inconsistent with option
pricing models as well as term structure models for implied volatility.

v. Base the calculation of the VIX on variance swap or "fair" volatility. Fair volatility is
typically quite close to At-The-Money implied volatility (typically greater by a factor of 1.07 with
a range of 1.02 to 1.12 for one month options). The major advantages of moving the calculation
to fair volatility from ATM implied volatility are:

a. Products based on the square of the fair volatility are hedgeable using an appropriately
weighted static strip of options (options at different strikes, but the same maturity). This
automatically means that options market makers will be in a better position to make markets
on the VIX.

b. Fair volatility is calculated from option prices rather than implied volatility. Therefore the
assumptions used in calculating implied volatility (in particular the option pricing model) are
not required to calculate fair volatility.

c. Fair volatility is calculated using a range of options prices at different strikes. Therefore fair
volatility captures the skew, and is also less dependent on individual ATM options prices.

We believe that fair volatility can be approximated with quite a simple formula -- see Appendix B
for the details. While we appreciate that fair volatility is a more complex concept than ATM
implied volatility, we believe that producing an index which can be easily hedged (or at least
whose square can be hedged) has strong advantages over other formulations of the VIX. We also
note that users of the VIX have accepted the current calculation with its rather unusual
relationship to quoted implied volatility and that the current calculations itself complex. Users
may not be particularly concerned about the nuances of fair volatility to ATM implied volatility.
Finally we believe that having one traded product (based on the VIX)$^3$ is a much neater solution
than having two products, one based on the current VIX, widely followed, but difficult to hedge,
and the other based on variance swaps, easier to hedge but with relatively lower following.

vi. Make any products based on the VIX based on the square of the index. Products based on
volatility (standard deviation) are extremely difficult to hedge. In contrast, products based on
variance can be hedged with a static strip of options. The best definition of variance for hedging
purposes is fair variance, as described in Appendix B. However, even if it is not possible to use
the fair variance calculation, based on the square of ATM implied volatility would be substantially
easier to hedge than products based on ATM implied volatility itself.

We believe that each of these modifications would improve the likelihood of success of contracts based on
the VIX index. Please do contact us if we can provide further details or clarify in any way.

Best regards,

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Appendix A: Relationship of current VIX volatility to one month ATM implied volatility due to calendar day adjustment

If implied volatility on the OEX ATM nearby and second nearby options were fixed, the current VIX calculation would cause the VIX to have a level above OEX ATM volatility by a factor illustrated below. This is a result of the calendar day adjustment.
Appendix B: Calculation of Variance Swap, or “Fair” Volatility

\[ \sigma_{var}^2 = \frac{2}{T} \sum_i \frac{\Delta K}{K_i} e^{rT} P_{GTM}(K_i) - \frac{1}{T} \left( \frac{e^{(r-d)T}S}{K_0} - 1 \right)^2 \]

Where:

- \( \sigma_{var} \) is the fair volatility
- \( T \) is the time to maturity (in years - i.e. calendar days / 365)
- \( S \) is the current index level
- \( \Delta K \) is the spacing between strikes used for calculation
- \( K_i \) is the strike price for option \( i \)
- \( K_0 \) is the first option strike below the current index level \( S \)
- \( r \) is the interest rate to time \( T \)
- \( P_{GTM}(K_i) \) is the quoted mid-market price (not volatility) for out-of-the-money option at strike \( K_i \) (i.e. a call for strikes \( > \) \( K_0 \), a put for strikes \( < \) \( K_0 \)). At \( K_0 \) use average of put and call prices.

Given \( \sigma_{var} \) calculated for options at the nearby \( (T_1) \) and the second nearby \( (T_2) \) option terms, then we can linearly interpolate in time \( t \) \( \sigma_{var,t}^2 \) to calculate \( \sigma_{var,30/60/90}^2 \).

This calculation is an approximation for true fair volatility, but we believe it is quite a close approximation. We would recommend using a total of eight strike prices (four puts and four calls), currently with a separation of 25 index points.